

Unitarity and Lee-Wick prescription at one loop level in Myers-Pospelov electrodynamics: the $e^+ + e^-$ annihilation

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We study perturbative unitarity in a Lorentz symmetry violating QED model with higher-order derivative operators in the light of the results of Lee and Wick to preserve unitarity in indefinite metric theories. Specifically, we consider the fermionic sector of the Myers-Pospelov model, which includes dimension five operators, coupled to standard photons. We canonically quantize the model and show that its Hamiltonian is stable, emphasizing the exact stage at which the indefinite metric appears and decomposes into a positive-metric sector and negative-metric sector. Finally, we verify the optical theorem at the one-loop level in the annihilation channel of the forward scattering process $e^+(p_2, r) + e^-(p_1, s)$ by applying the Lee-Wick prescription in which the states associated with the negative metric are left out from the asymptotic Hilbert space.

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I. INTRODUCTION

Gravitational effects of elementary particles are expected to become significant at energy scales of the Planck mass $m_P \approx 10^{19}$ GeV. To describe the interplay between gravity and matter at these energies and to search for new physics, an effective approach has been actively exploited in the absence of a more fundamental theory. A class of gravitationally induced effects which could be observable at standard model energies is the breakdown of Lorentz symmetry, which nevertheless is Planck-mass suppressed. Many experiments have been designed to possibly detect such weak signals and they range from precision laboratory experiments to astrophysical observations.

The Standard-Model Extension (SME) is an effective framework that incorporates all possible Lorentz-invariance violating terms for matter and gravity in the Lagrangian. The breakdown of Lorentz symmetry originates from preferred directions which are believed to arise from expectation values of tensor fields in a more fundamental theory. A great number of the phenomenological searches for Lorentz symmetry violation has been codified within the framework of the SME [1, 2]. Originally it was constructed to include only renormalizable mass-dimension operators, i.e, with dimension $d \leq 4$. Recently, a generalization of the SME incorporating higher-order derivative operators has been proposed. Such program has been successfully implemented in the photon sector [3], fermion sector [4], and more recently in the linearized sector of gravity [5].

The pioneering work of Myers and Pospelov focusses on Lorentz-invariance violation with dimension-five operators coupled to a constant four-vector n_μ and having

cubic dispersion relations in the lowest order momentum expansion [6]. The Myers-Pospelov (MP) model has been studied to extract bounds upon its parameters from radiative corrections [7–10], cosmological observations [11], anisotropies [12], synchrotron radiation [13] and also to analyze stability and causality [14]. One can show that for a special choice of nonminimal SME coefficients one arrives to the MP model. Recently, an approach to introduce higher-order Lorentz symmetry violation which lies beyond the scope of the nonminimal SME with modified terms quadratic in the fields, has been proposed with higher-order coupling terms [15].

The interest in higher-order derivative operators in quantum field theory dates back to the work of Podolsky [16]. He considered a higher-order electrodynamics to deal with infinities arising from the introduction of point charges. Some years later Pais and Uhlenbeck realized that these higher-order derivative terms may lead to some problems with stability [17]. The breakthrough in relation to stability came with the studies of Lee and Wick in the context of quantum field theories with indefinite metric [18]. They give an important insight into the relation between the possible loss of unitarity and the interplay between statistics, stability and negative norm-states. Recently, the Lee-Wick ideas have been applied to solve the hierarchy problem in the standard model [19], to study the spectrum of cosmological perturbations [20] and to construct a renormalization program in higher derivative gravity [21]. Also, higher-order derivative operators have been included in quantum gravity approaches [22, 23], in anisotropic regularization schemes [24], in semiclassical gravity [25] and arise in the study of the phenomenology of loop quantum gravity [26] and in string theory [27].

In 1969, T. D. Lee and G. W. Wick proposed a modified QED model with the advantage of being finite, but leading to an indefinite metric in Hilbert space [18]. They provide the main ideas towards the construction of an indefinite-metric quantum field theory with a uni-

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tary S -matrix. The indefiniteness of the metric in the Lee-Wick quantum electrodynamics comes from a non-hermitian Hamiltonian which however can be seen to arise from the presence of a higher-order derivative term as well [28]. Several issues regarding stability and unitarity were solved using what is now called the Lee-Wick prescription. The analysis was extended by Cutkosky using covariant perturbation theory based on Feynman diagrams [29].

The origin of the possible loss of unitarity in an indefinite metric theory can be found in the definition of the inner product. To see this, consider two arbitrary states $|\phi\rangle = \sum_i \phi_i |i\rangle$ and $|\psi\rangle = \sum_j \psi_j |j\rangle$ expanded in a basis $|i\rangle$ with $i = 1, 2, 3, \dots$ and ϕ_i, ψ_j complex numbers. As in usual quantum mechanics the inner product between the two states is defined by $\langle\phi|\psi\rangle = \phi_i^* \eta_{ij} \psi_j$, where the metric $\eta = (\eta_{ij})$ is assumed to be a nonsingular Hermitian matrix and where the asterisk denotes complex conjugation. Now, however, the generalization consists to allow for an indefinite metric, such that the diagonal terms of $\eta_{ij} = \langle i|j\rangle$ can take negative values. In this way, the metric η in the Hilbert space is not positive definite and one may have states with negative norm or ghosts in the theory. The extended inner product induced by the indefinite metric η in general leads to a pseudo-unitary condition for the S -matrix, i.e., $S^\dagger \eta S = \eta$. However, it was shown by Lee-Wick that by removing the negative-metric particles from the asymptotic observable states of the theory and defining a suitable choice for the position of the poles in each Feynman diagram, the unitarity of the S -matrix can be preserved [30, 31]. Importantly, one can show that in an indefinite metric theory the algebra of creation and annihilation operators completely determines the class of metrics η . For example in a fermion system, once the Lagrangian is given, the equal-time anti-commutation relations lead to a unique metric representation. However, for bosons using a redefinition of the vacuum state one may change from an indefinite metric representation to a positive definite metric representation, which however may lead to instabilities.

Studies on perturbative unitarity in Lorentz violating theories have been carried out in the renormalizable sector [32] and nonrenormalizable sector [33]. At tree level the conservation of energy plays a key role to verify unitarity when higher-order derivative operators are present [33]. However, the preservation of unitarity is more involved when virtual particles are created, as in loop diagrams, since then, one has to consider the discontinuities of the poles associated to the particles with negative-metric as well. In this work, using the ideas of Lee and Wick to deal with indefinite metric theories, we extend some previous studies on unitarity to the one-loop level [34].

We consider the timelike MP model, where only the fermionic sector is modified with respect to standard QED, introducing higher-order time derivative operators yielding Lorentz symmetry violation. In Section II we provide the construction of the free fermionic field of the

model, together with their basic properties: the calculation of the dispersion relations, the definition of the corresponding creation-annihilation operators, the verification of the anticommutation rules for the canonically conjugated field variables, the construction of the Hamiltonian and the derivation of the free field propagator using two different approaches: the vacuum expectation value of the time ordered product $\psi(x)\bar{\psi}(y)$ and the integration over momentum space. We follow closely the conventions and results of Ref. [35].

In section III we compute the imaginary part of the amplitude for the one-loop diagram in the annihilation channel arising from the forward scattering process $e^+(p_2, r) + e^-(p_1, s)$. The calculation is made using slightly modified Feynman rules with respect to Ref. [35], which are stated at the beginning of this section. To this end we calculate the amplitude \mathcal{M}_F for the corresponding graph and we also identify the integral $J_{\mu\nu}$ that produces the discontinuity in the amplitude \mathcal{M}_F , which yields the corresponding imaginary part. The contributions to such integral are determined by the method of residues according to the appropriately defined Lee-Wick contour, which is constructed by taking the same position of the poles which yields the correct answer when the propagator is calculated by integrating in momentum space.

The discontinuities of \mathcal{M}_F arising from $J_{\mu\nu}$ are subsequently obtained yielding some unexpected cancellations, which nevertheless are crucial to prove the validity of the optical theorem in this case. Some details in the derivation of such discontinuities are given in the Appendix. In section IV we determine the amplitude \mathcal{M}_I for the process $e^+(p_2, r) + e^-(p_1, s) \rightarrow e^+(k_2, \bar{r}) + e^-(k_1, \bar{s})$ and calculate the sum over the momenta and spins of the final states in $|\mathcal{M}_I|^2$ as required by the optical theorem. Unitarity is successfully verified by comparing this result with that obtained in the previous section for the imaginary part of \mathcal{M}_F . We close with section V which contain our conclusions and final comments.

II. MODEL DEFINITIONS

The modified QED Lagrangian we are interested in is obtained via the minimal coupling substitution in the derivative terms of the fermionic sector in the Myers and Pospelov (MP) model,

$$\mathcal{L} = \bar{\psi}(i\bar{D} - m)\psi + g\bar{\psi}\not{n}(n \cdot D)^2\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}, \quad (1)$$

where $D_\mu = \partial_\mu - ieA_\mu$ and n_μ is a constant four vector breaking the Lorentz symmetry and chosen in the time-like direction, such that $n_\mu = (1, 0, 0, 0)$. As usual, we will quantize this extended electrodynamics in the interaction picture and we follow the conventions of Ref. [35]. We will work in the axial gauge $(n \cdot A) = 0$, such that the interaction term is given by the standard one in QED. Also, the photon properties remain the same as in QED.

On the contrary, the fermion sector will be drastically modified and we start from the study of their properties in the free case. The corresponding equation of motion is

$$(i\cancel{\partial} - m + g\gamma^0\partial_0^2)\psi(x) = 0, \quad (2)$$

which includes higher-order time derivatives. Considering $\psi(x) = \int dp \psi(p)e^{-ip \cdot x}$ we obtain the eigenvalue equation for the spinor field $\psi(p)$,

$$(\gamma^0(p_0 - gp_0^2) + \gamma^i p_i - m)\psi(p) = 0. \quad (3)$$

Taking the determinant of the above matrix yields the dispersion relation

$$(p_0 - gp_0^2)^2 - \mathbf{p}^2 - m^2 = 0, \quad (4)$$

whose solutions are,

$$\begin{aligned} \omega_1 &= \frac{1 - \sqrt{1 - 4gE(\mathbf{p})}}{2g} = \frac{1 - N_1}{2g}, \\ W_1 &= \frac{1 + \sqrt{1 - 4gE(\mathbf{p})}}{2g} = \frac{1 + N_1}{2g}, \end{aligned} \quad (5)$$

together with

$$\begin{aligned} \omega_2 &= \frac{1 - \sqrt{1 + 4gE(\mathbf{p})}}{2g} = \frac{1 - N_2}{2g}, \\ W_2 &= \frac{1 + \sqrt{1 + 4gE(\mathbf{p})}}{2g} = \frac{1 + N_2}{2g}. \end{aligned} \quad (6)$$

Here $E(\mathbf{p}) = \sqrt{\mathbf{p}^2 + m^2}$, $N_1 = \sqrt{1 - 4gE(\mathbf{p})}$ and $N_2 = \sqrt{1 + 4gE(\mathbf{p})}$. Let us observe that these functions are invariant under the change $\mathbf{p} \rightarrow -\mathbf{p}$. We identify the solutions ω_1 and ω_2 with perturbations in g of the usual ones $\pm E(\mathbf{p})$, while W_1 and W_2 correspond to the contributions of new degrees of freedom coming from higher-energy scales, which are non-perturbative in g . We emphasize that ω_2 is negative, so that the energy corresponding to this on-shell particle is $|\omega_2| = -\omega_2$. The above eigenvalues satisfy the relation

$$W_1 + \omega_1 = W_2 + \omega_2. \quad (7)$$

The following additional identities follows from the definitions (5) and (6)

$$\begin{aligned} E(\mathbf{p}) &= \omega_1 - g\omega_1^2, \\ E(\mathbf{p}) &= W_1 - gW_1^2, \\ -E(\mathbf{p}) &= \omega_2 - g\omega_2^2, \\ -E(\mathbf{p}) &= W_2 - gW_2^2, \end{aligned} \quad (8)$$

together with

$$\begin{aligned} E(\mathbf{p}) - g\omega_1^2 &= \omega_1 N_1, \\ E(\mathbf{p}) - gW_1^2 &= -W_1 N_1, \\ E(\mathbf{p}) + g\omega_2^2 &= -\omega_2 N_2, \\ E(\mathbf{p}) + gW_2^2 &= W_2 N_2. \end{aligned} \quad (9)$$

From Eq. (8) we observe that $(p_0 - gp_0^2) = +E(\mathbf{p})$ for $p_0 = \omega_1$ or $p_0 = W_1$ while $(p_0 - gp_0^2) = -E(\mathbf{p})$ for $p_0 = \omega_2$ or $p_0 = W_2$. In this way, the corresponding spinors satisfy the Dirac equations

$$(\gamma^0 E(\mathbf{p}) + \gamma^i p_i - m)u(\mathbf{p}) = 0, \text{ for } p_0 = \omega_1, W_1, \quad (10)$$

and

$$(\gamma^0 E(\mathbf{p}) + \gamma^i p_i + m)v(\mathbf{p}) = 0, \text{ for } p_0 = \omega_2, W_2, \quad (11)$$

which correspond to the standard spinor solutions $u^r(\mathbf{p})$ and $v^s(\mathbf{p})$ labelled with the spin index r, s . Our conventions for the completeness relations are

$$\begin{aligned} \sum_r u^r(\mathbf{p})\bar{u}^r(\mathbf{p}) &= \cancel{\not{p}} + m, \\ \sum_{r'} v^{r'}(\mathbf{p})\bar{v}^{r'}(\mathbf{p}) &= \cancel{\not{p}} - m, \end{aligned} \quad (12)$$

while for orthogonality we have

$$\begin{aligned} u^{s\dagger}(\mathbf{p})u^r(\mathbf{p}) &= 2E_p\delta^{sr}, \\ v^{s\dagger}(\mathbf{p})v^r(\mathbf{p}) &= 2E_p\delta^{sr}, \\ u^{s\dagger}(\mathbf{p})v^r(-\mathbf{p}) &= 0, \\ v^{s\dagger}(\mathbf{p})u^r(-\mathbf{p}) &= 0. \end{aligned} \quad (13)$$

The above relations can be equivalently written as

$$\begin{aligned} \bar{u}^s(\mathbf{p})u^r(\mathbf{p}) &= 2m\delta^{sr}, \\ \bar{v}^s(\mathbf{p})v^r(\mathbf{p}) &= -2m\delta^{sr}, \\ \bar{u}^s(\mathbf{p})v^r(\mathbf{p}) &= 0, \\ \bar{v}^s(\mathbf{p})u^r(\mathbf{p}) &= 0. \end{aligned} \quad (14)$$

The fields are expanded in terms of their eigenfunctions as

$$\begin{aligned}\psi(x) &= \sum_s \int \frac{d^3p}{\sqrt{2E(\mathbf{p})(2\pi)^3}} \left(\frac{u^s(\mathbf{p})}{\sqrt{N_1}} (a_p^s e^{-i\omega_1 x_0} + c_p^s e^{-iW_1 x_0}) e^{i\mathbf{p}\cdot\mathbf{x}} + \frac{v^s(\mathbf{p})}{\sqrt{N_2}} (b_p^{s\dagger} e^{-i\omega_2 x_0} + d_p^s e^{-iW_2 x_0}) e^{-i\mathbf{p}\cdot\mathbf{x}} \right), \\ \bar{\psi}(x) &= \sum_s \int \frac{d^3p}{\sqrt{2E(\mathbf{p})(2\pi)^3}} \left(\frac{\bar{u}^s(\mathbf{p})}{\sqrt{N_1}} (a_p^{s\dagger} e^{i\omega_1 x_0} + c_p^{s\dagger} e^{iW_1 x_0}) e^{-i\mathbf{p}\cdot\mathbf{x}} + \frac{\bar{v}^s(\mathbf{p})}{\sqrt{N_2}} (b_p^s e^{i\omega_2 x_0} + d_p^{s\dagger} e^{iW_2 x_0}) e^{i\mathbf{p}\cdot\mathbf{x}} \right).\end{aligned}\quad (15)$$

The commutation relations for particles and antiparticles are taken to be the usual

$$\{a_p^s, a_q^{r\dagger}\} = \{b_p^s, b_q^{r\dagger}\} = (2\pi)^3 \delta^{sr} \delta(\mathbf{p} - \mathbf{q}), \quad (16)$$

but for the new particles we have

$$\{c_p^s, c_q^{r\dagger}\} = \{d_p^s, d_q^{r\dagger}\} = -(2\pi)^3 \delta^{sr} \delta(\mathbf{p} - \mathbf{q}). \quad (17)$$

Their action on the vacuum is defined by

$$\begin{aligned}a_p^s|0\rangle &= 0, & b_p^s|0\rangle &= 0, \\ c_p^s|0\rangle &= 0, & d_p^s|0\rangle &= 0.\end{aligned}\quad (18)$$

The anticommutation relations in Eqs. (16) and (17) display the exact stage at which the indefinite metric decomposes into a positive-metric sector and negative-metric sector. Of course, due to the negative sign in Eq. (17) one is led to identify the particles created with the operators $c_q^{r\dagger}$ and $d_q^{r\dagger}$ as those with negative-metric while those created with $a_q^{r\dagger}$ and $b_q^{r\dagger}$ as particles with positive-metric. We will show in the subsection (II A) that Eqs. (16) and (17) are necessary in order to fulfill the equal-time anticommutation relations given in Eq. (23).

The choice of vacuum in Eq. (18) leads to the usual interpretation of the field $\psi(x)$ annihilating fermions with positive energy ω_1 and creating anti-fermions with positive energy $|\omega_2|$. In addition, the field $\psi(x)$ annihilates negative-metric fermions with positive energy W_1 and negative-metric anti-fermions with positive energy W_2 . We will show in subsection (II B) that this choice of vacuum leads to a Hamiltonian bounded from below.

A. Canonical variables

Here we deal with the canonical quantization for the purely timelike MP Lagrangian

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi + g\psi^\dagger\ddot{\psi}. \quad (19)$$

The canonically conjugated momenta to $\dot{\psi}$

$$\pi_\psi = \frac{\partial\mathcal{L}}{\partial\dot{\psi}}, \quad (20)$$

and to ψ

$$\pi_\psi = \frac{\partial\mathcal{L}}{\partial\dot{\psi}} - \frac{\partial\pi_{\dot{\psi}}}{\partial t}, \quad (21)$$

are given by

$$\begin{aligned}\pi_\psi &= i\psi^\dagger - g\dot{\psi}^\dagger, \\ \pi_{\dot{\psi}} &= g\psi^\dagger.\end{aligned}\quad (22)$$

Starting from the relations (15), (16) and (17) we have verified the following equal-time commutation relations

$$\begin{aligned}\{\psi(x), \pi_\psi(y)\} &= i\delta^3(\mathbf{x} - \mathbf{y}), \\ \{\dot{\psi}(x), \pi_{\dot{\psi}}(y)\} &= i\delta^3(\mathbf{x} - \mathbf{y}),\end{aligned}\quad (23)$$

with the remaining ones being zero.

B. Stability

The Hamiltonian density is obtained from the Legendre transformation $\mathcal{H} = \pi_\psi\dot{\psi} + \pi_{\dot{\psi}}\ddot{\psi} - \mathcal{L}$, leading to the Hamiltonian

$$H = \int d^3x \left(-g\dot{\psi}^\dagger(x)\dot{\psi}(x) + \bar{\psi}(x)(-i\gamma^i\partial_i + m)\psi(x) \right). \quad (24)$$

In a calculation analogous to the standard fermionic case we have verified that the Hamiltonian can be written as

$$\begin{aligned}H &= \sum_s \int \frac{d^3p}{(2\pi)^3} (\omega_1 a_p^{s\dagger} a_p^s + \omega_2 b_p^s b_p^{s\dagger} - W_1 c_p^{s\dagger} c_p^s \\ &\quad - W_2 d_p^{s\dagger} d_p^s),\end{aligned}\quad (25)$$

in terms of the creation-annihilation operators. In obtaining Eq. (25), the identities (9) have been repeatedly used. Since $\omega_2 < 0$ and $\omega_1 > 0$, the above Eq. (25) provides the usual interpretation for particle and antiparticles states: $a_p^{s\dagger}$ creates particles with momentum \mathbf{p} , spin component s and energy $\omega_1(\mathbf{p})$ and $b_p^{s\dagger}$ creates antiparticles with momentum \mathbf{p} , spin component s and energy $|\omega_2(\mathbf{p})|$, both of which are observable. Similar interpretation is given for the operators $c_p^{s\dagger}$ and $d_p^{s\dagger}$ in terms of particles described by states with negative-metric.

Introducing the number operators for particles in states with positive metric $\hat{N}_{1p} = \sum_s a_p^{s\dagger} a_p^s$, $\hat{N}_{2p} = \sum_s b_p^{s\dagger} b_p^s$ and for particles in states with negative metric $\hat{N}_{1p} = -\sum_s c_p^{s\dagger} c_p^s$, $\hat{N}_{2p} = -\sum_s d_p^{s\dagger} d_p^s$, we can write

$$H = \int \frac{d^3p}{(2\pi)^3} \left(\omega_1 \hat{N}_{1p} - \omega_2 \hat{N}_{2p} + W_1 \hat{N}_{1p} + W_2 \hat{N}_{2p} \right), \quad (26)$$

which is clearly bounded from below after dropping the usual infinite constant.

C. The Propagator

Here we derive the fermion propagator $S_F(x - y)$ for the purely timelike MP model defined by the Lagrangian (19). To verify that its four-momentum representation (where one specifies the position of the poles $\omega_1, \omega_2, W_1, W_2$ in the complex p_0 -plane and hence the contour integration C_F) is correct, we first calculate the

propagator according to its definition in terms of the vacuum expectation value $S_F(x - y) = \langle 0|T \{ \psi(x) \bar{\psi}(y) \} |0 \rangle$, yielding

$$S_F(x - y) = \theta(x_0 - y_0) \langle 0| \psi(x) \bar{\psi}(y) |0 \rangle - \theta(y_0 - x_0) \langle 0| \bar{\psi}(y) \psi(x) |0 \rangle. \quad (27)$$

Without loss of generality, we can set $y = 0$. First, we consider $x_0 > 0$, and hence we need to calculate $S_F^{(>)}(x) = \langle 0| \psi(x) \bar{\psi}(0) |0 \rangle$. Using the expressions for the fields in Eq. (15) we find

$$S_F^{(>)}(x) = \langle 0| \left[\int \frac{d^3 p}{\sqrt{2E(\mathbf{p})(2\pi)^3}} \sum_s \left(\frac{a_p^s u^s(p)}{\sqrt{N_1}} e^{-i\omega_1 x_0 + i\mathbf{p} \cdot \mathbf{x}} + \frac{c_p^s u^s(p)}{\sqrt{N_1}} e^{-iW_1 x_0 + i\mathbf{p} \cdot \mathbf{x}} + \frac{d_p^s v^s(p)}{\sqrt{N_2}} e^{-iW_2 x_0 - i\mathbf{p} \cdot \mathbf{x}} \right) \right. \\ \left. \times \int \frac{d^3 p'}{\sqrt{2E(\mathbf{p}')(2\pi)^3}} \sum_r \left(\frac{a_{p'}^{r\dagger} \bar{u}^r(p')}{\sqrt{N_1'}} + \frac{c_{p'}^{r\dagger} \bar{u}^r(p')}{\sqrt{N_1'}} + \frac{d_{p'}^{r\dagger} \bar{v}^r(p')}{\sqrt{N_2'}} \right) \right] |0 \rangle. \quad (28)$$

The properties in Eq.(18) allow us to rewrite $\langle 0| X_p^s X_q^{r\dagger} |0 \rangle = \langle 0| \{ X_p^s, X_q^{r\dagger} \} |0 \rangle = \pm (2\pi)^3 \delta^{rs} \delta^3(\mathbf{p} - \mathbf{q})$,

where the plus sign is for $X = a, b$ and the minus sign for $X = c, d$. This yields

$$S_F^{(>)}(x) = \int \frac{d^3 p}{2E(\mathbf{p})(2\pi)^3} \sum_s \left(\frac{u^s(p) \bar{u}^s(p)}{N_1} (e^{-i\omega_1 x_0 + i\mathbf{p} \cdot \mathbf{x}} - e^{-iW_1 x_0 + i\mathbf{p} \cdot \mathbf{x}}) - \frac{v^s(p) \bar{v}^s(p)}{N_2} e^{-iW_2 x_0 - i\mathbf{p} \cdot \mathbf{x}} \right). \quad (29)$$

Using the completeness relation in Eq. (12) we obtain

$$S_F^{(>)}(x) = \int \frac{d^3 p}{2E(\mathbf{p})(2\pi)^3} \left(\frac{\not{p} + m}{N_1} (e^{-i\omega_1 x_0 + i\mathbf{p} \cdot \mathbf{x}} - e^{-iW_1 x_0 + i\mathbf{p} \cdot \mathbf{x}}) - \frac{\not{p} - m}{N_2} e^{-iW_2 x_0 - i\mathbf{p} \cdot \mathbf{x}} \right), \quad (30)$$

which can be rewritten as

$$S_F^{(>)}(x) = (i\not{\partial} + m + g\gamma^0 \partial_0^2) \int \frac{d^3 p}{2E(\mathbf{p})(2\pi)^3} \left(\frac{e^{-i\omega_1 x_0 + i\mathbf{p} \cdot \mathbf{x}}}{N_1} - \frac{e^{-iW_1 x_0 + i\mathbf{p} \cdot \mathbf{x}}}{N_1} + \frac{e^{-iW_2 x_0 - i\mathbf{p} \cdot \mathbf{x}}}{N_2} \right). \quad (31)$$

In analogous way, for $x_0 < 0$, we find

$$S_F^{(<)}(x) = - \int \frac{d^3 p}{2E(\mathbf{p})(2\pi)^3} \left(\frac{\not{p} - m}{N_2} e^{-i\omega_2 x_0 - i\mathbf{p} \cdot \mathbf{x}} \right), \quad (32)$$

which can be cast in the same form as Eq. (31)

$$S_F^{(<)}(x) = (i\not{\partial} + m + g\gamma^0 \partial_0^2) \int \frac{d^3 p}{2E(\mathbf{p})(2\pi)^3} \frac{e^{-i\omega_2 x_0 - i\mathbf{p} \cdot \mathbf{x}}}{N_2}. \quad (33)$$

Now we turn to the calculation of the propagator in momentum space. From the equation of motion (3) the

Feynman propagator is

$$S_F(x - y) = \int \frac{d^3 \mathbf{p} dp_0}{(2\pi)^4} \frac{i(\not{P} + m)}{P^2 - m^2} e^{-i\mathbf{p} \cdot (x - y)} \quad (34)$$

where $P^\mu = (p_0 - gp_0^2, \mathbf{p})$. The propagator in momentum space is

$$S(p) = \frac{i(\not{P} + m)}{P^2 - m^2}, \quad (35)$$

where one has to specify the position of the poles arising from the dispersion relations (4), with solutions (5) and (6), in the complex p_0 plane. The standard $P^2 - m^2 +$

$i\epsilon$ prescription yields the following choice for the poles: $p_0 = \omega_1 - i\epsilon$, $p_0 = W_1 + i\epsilon$, $p_0 = \omega_2 + i\epsilon$, $p_0 = W_2 - i\epsilon$. Nevertheless, in order to reproduce the expressions (31) and

(33) for the propagator we have to choose the position for the corresponding poles as depicted in the following denominator of $S(p)$

$$S(p) = \frac{i(\not{p} + m)}{g^2(p_0 - (\omega_1 - i\epsilon))(p_0 - (\omega_2 + i\epsilon))(p_0 - (W_1 - i\epsilon))(p_0 - (W_2 - i\epsilon))}. \quad (36)$$

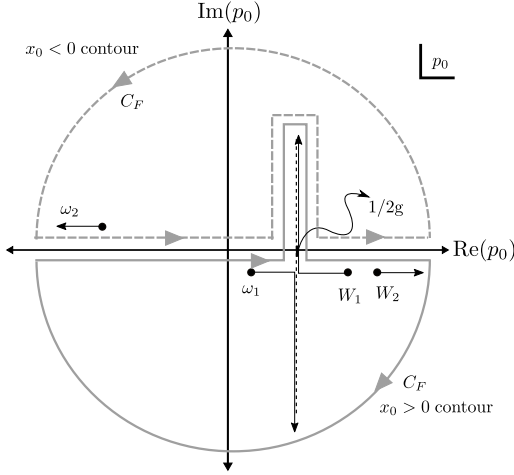


FIG. 1: The contour of integration C_F which defines the Feynman propagator. For $x_0 > 0$, it picks up the positive poles ω_1, W_1, W_2 and for $x_0 < 0$ it picks up the negative pole ω_2 . At the energy $E = 1/4g$ the two poles ω_1 and W_1 collide and move in the opposite direction parallel to the imaginary axis. The poles ω_2 and W_2 always stay in the real axis and move in opposite directions.

That is to say, ω_1, W_1 and W_2 are in the lower p_0 complex plane, while ω_2 is in the upper p_0 complex plane. This choice is shown in Fig. 1, together with the motion of the poles as momenta \mathbf{p} increases. The arrows indicate their trajectories in the p_0 plane according to Eqs. (5) and (6). We see that both poles ω_1 and W_1 collapse at the value $1/2g$ when $E(\mathbf{p}) = 1/4g$ and move in the opposite imaginary direction when $E(\mathbf{p}) > 1/4g$. As $|\mathbf{p}|$ increases, the poles ω_2 and W_2 move in the real and opposite directions. In this way, the Feynman contour C_F is defined as the real axis with the poles located as shown in Fig. 1.

Next we derive the propagator using the expression (36) together with the contour C_F and show that we recover the expressions (31) and (33) obtained in the previous calculation by using its definition in terms of vacuum expectation values. To this end, let us set $y = 0$ and consider $x_0 > 0$. The factor $e^{ip_0 x_0}$ in Eq. (34) indicates that we have to close our contour from below ($\text{Im } p_0 < 0$),

thus enclosing the poles ω_1, W_1 and W_2 . This yields

$$S_F^{(>)}(x) = i \int \frac{d^3 p}{(2\pi)^4} (-2\pi i) \left((\gamma^0 E(\mathbf{p}) - \gamma \cdot \mathbf{p} + m) \times \left(\frac{e^{-i\omega_1 x_0 + i\mathbf{p} \cdot \mathbf{x}}}{g^2(\omega_1 - \omega_2)(\omega_1 - W_1)(\omega_1 - W_2)} \right. \right. \\ \times \left. \frac{e^{-iW_1 x_0 + i\mathbf{p} \cdot \mathbf{x}}}{g^2(W_1 - \omega_1)(W_1 - \omega_2)(W_1 - W_2)} \right) \\ \left. + (-\gamma^0 E(\mathbf{p}) - \gamma \cdot \mathbf{p} + m) \times \frac{e^{-iW_2 x_0 + i\mathbf{p} \cdot \mathbf{x}}}{g^2(W_2 - \omega_1)(W_2 - \omega_2)(W_2 - W_1)} \right). \quad (37)$$

Using the identities

$$\begin{aligned} g^2(\omega_1 - \omega_2)(\omega_1 - W_1)(\omega_1 - W_2) &= 2EN_1, \\ g^2(W_1 - \omega_1)(W_1 - \omega_2)(W_1 - W_2) &= -2EN_1, \\ g^2(W_2 - \omega_1)(W_2 - \omega_2)(W_2 - W_1) &= 2EN_2, \end{aligned} \quad (38)$$

we have

$$S_F^{(>)}(x) = \int \frac{d^3 p}{(2\pi)^3} \left((\gamma^0 E(\mathbf{p}) - \gamma \cdot \mathbf{p} + m) \times \left(\frac{e^{-i\omega_1 x_0 + i\mathbf{p} \cdot \mathbf{x}}}{2EN_1} - \frac{e^{-iW_1 x_0 + i\mathbf{p} \cdot \mathbf{x}}}{2EN_1} \right) \right. \\ \left. + (-\gamma^0 E(\mathbf{p}) + \gamma \cdot \mathbf{p} + m) \frac{e^{-iW_2 x_0 - i\mathbf{p} \cdot \mathbf{x}}}{2EN_2} \right), \quad (39)$$

where we have changed $\mathbf{p} \rightarrow -\mathbf{p}$ in the last term. This is the same as

$$S_F^{(>)}(x) = (i\not{\partial} + m + g\gamma^0 \partial_0^2) \int \frac{d^3 p}{2E(2\pi)^3} \left(\frac{e^{-i\omega_1 x_0 + i\mathbf{p} \cdot \mathbf{x}}}{N_1} - \frac{e^{-iW_1 x_0 + i\mathbf{p} \cdot \mathbf{x}}}{N_1} + \frac{e^{-iW_1 x_0 - i\mathbf{p} \cdot \mathbf{x}}}{N_2} \right). \quad (40)$$

The above expression reproduces the form of the propagator obtained in Eq. (31).

When $x_0 < 0$, the p_0 integration is made by closing the contour from above, and we obtain

$$S_F^{(<)}(x) = i \int \frac{d^3 p}{(2\pi)^4} (2\pi i) \left((-\gamma^0 E(\mathbf{p}) - \gamma \cdot \mathbf{p} + m) \times \frac{e^{-i\omega_2 x_0 + i\mathbf{p} \cdot \mathbf{x}}}{g^2(\omega_2 - \omega_1)(\omega_2 - W_1)(\omega_2 - W_2)} \right). \quad (41)$$

Using now the relation

$$g^2(\omega_2 - \omega_1)(\omega_2 - W_1)(\omega_2 - W_2) = -2EN_2, \quad (42)$$

we finally arrive to

$$S_F^{(<)}(x) = (i\cancel{\partial} + m + g\gamma^0\partial_0^2) \int \frac{d^3p}{2E(2\pi)^3} \frac{e^{-i\omega_2 x_0 - i\mathbf{p}\cdot\mathbf{x}}}{N_2}, \quad (43)$$

which reproduces Eq. (33). In this way, we have proved that the prescription (36) for the position of the poles in the complex p_0 plane yields the correct fermionic propagator, according to the definition in Eq. (27).

III. UNITARITY IN THE MYERS-POSPELOV ELECTRODYNAMICS: ONE LOOP LEVEL

Our goal is to verify the optical theorem for the simple diagram shown in Fig. 2. In the conventions of Ref. [35] this theorem reads

$$2\text{Im}\mathcal{M}(A \rightarrow A) = \sum_n \prod_i^n \int \sum_{\bar{s}_i} \left(\frac{d^3k_i}{(2\pi)^3} \frac{1}{2E(\mathbf{k}_i)} \right) \times |\mathcal{M}(A \rightarrow B(\mathbf{k}_i, \bar{s}_i))|^2 (2\pi)^4 \delta^4 \left(p_A - \sum_i \mathbf{k}_i \right). \quad (44)$$

Here A denotes the initial and final processes associated to the amplitude $\mathcal{M}(A \rightarrow A) \equiv \mathcal{M}_F$, each carrying total momentum p_A . The term in the right hand side $\mathcal{M}(A \rightarrow B(\mathbf{k}_i, \bar{s}_i)) \equiv \mathcal{M}_I$ denotes the scattering amplitude for the process A going to the final states of n particles described by the process $B(\mathbf{k}_i, \bar{s}_i)$. In our case the process A corresponds to $e^-(p_1, s) + e^+(p_2, r)$, with $p_A = p_1 + p_2$ and the process B is $e^-(k_1, \bar{s}) + e^+(k_2, \bar{r})$.

We have adopted de Lee-Wick prescription where, the intermediate states in the right hand side of Eq. (44) include only the positive-metric states, i.e., e^- and e^+ in our case, while leaving out the negative-metric states [18]. Nevertheless, the latter states may contribute to the imaginary part of the left hand side via the propagator in the loop integral, and this mismatching could be a cause for the failure of unitarity. Still, as we show in the following, a proper choice of the so called Lee-Wick integration contour will restore unitarity at the level of the optical theorem (44).

We take the same definition and normalization of the one particle state as in Ref. [35], so that

$$\begin{aligned} |e^-, \mathbf{p}, s\rangle &= \sqrt{2E(\mathbf{p})} a_p^{s\dagger} |0\rangle, \\ |e^+, \mathbf{q}, r\rangle &= \sqrt{2E(\mathbf{q})} a_q^{r\dagger} |0\rangle, \end{aligned} \quad (45)$$

together with the normalization

$$\langle \mathbf{p}, s | \mathbf{q}, r \rangle = 2E(\mathbf{p})(2\pi)^3 \delta^{sr} \delta^3(\mathbf{p} - \mathbf{q}). \quad (46)$$

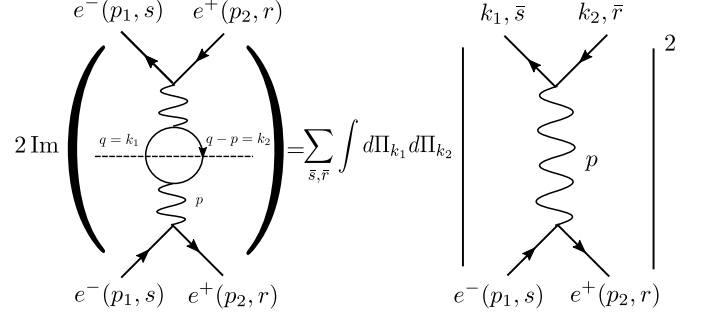


FIG. 2: The optical theorem, showing the forward scattering process $e^-(p_1, s) + e^+(p_2, r)$ in the left hand side and the sum over intermediate states $e^-(k_1, \bar{s}) + e^+(k_2, \bar{r})$, conserving total energy p_A , in the right hand side. The phase-space measure is defined by $\int d\Pi_{k_1} d\Pi_{k_2} = \int \left(\frac{d^3k_1}{(2\pi)^3} \frac{1}{2E(\mathbf{k}_1)} \right) \left(\frac{d^3k_2}{(2\pi)^3} \frac{1}{2E(\mathbf{k}_2)} \right)$ according to the usual normalization.

In this way the required Feynman rules to calculate the above amplitudes are the same as those for QED in Ref. [35], except for the following changes.

The photon propagator is

$$D_{\mu\nu}(p) = \frac{-i}{p^2 + i\epsilon} \left[\eta_{\mu\nu} - \frac{p_\mu n_\nu + n_\mu p_\nu}{(n \cdot p)} + p_\mu p_\nu \frac{n^2}{(n \cdot p)^2} \right], \quad (47)$$

which is given in the homogeneous temporal axial gauge. Nevertheless, in our case this propagator joins two conserved currents, in such a way that only the term proportional to $\eta_{\mu\nu}$ contributes. Also, we know that the Fadeev-Popov ghost required in the case of the axial gauges decouple, so that there is no contribution coming from them. Also, the fermion propagator is

$$S(q) = \frac{i(\gamma_0 f(q_0) + \gamma_i q^i + m)}{g^2(q_0 - \omega_1)(q_0 - \omega_2)(q_0 - W_1)(q_0 - W_2)}, \quad (48)$$

with the notation

$$f(q_0) = q_0 - gq_0^2, \quad (49)$$

in agreement with Eq. (35), where the position of the poles has been already specified in Eq. (36). Here we have contributions from the negative-metric states via the poles W_1 and W_2 . The last change in the Feynman rules comes from the fermionic external lines, where we have to introduce the following replacements

$$u^s(\mathbf{p}) \rightarrow \frac{u^s(\mathbf{p})}{N_1(\mathbf{p})}, \quad v^s(\mathbf{p}) \rightarrow \frac{v^s(\mathbf{p})}{N_2(\mathbf{p})}, \quad (50)$$

and analogously for $\bar{u}^s(\mathbf{p})$ and $\bar{v}^s(\mathbf{p})$. As established previously, the spinors $u^s(\mathbf{p})$ and $v^s(\mathbf{p})$ are the same as in

standard QED. The first relation in (50) can be directly seen from the action

$$\begin{aligned} \psi^+(x)|e^-, \mathbf{q}, r\rangle &= \sum_s \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E(\mathbf{p})}} \frac{u_s(\mathbf{p})}{\sqrt{N_1}} \\ &\times e^{-ip \cdot x} \sqrt{2E(\mathbf{q})} a_p^s a_q^{+r} |0\rangle, \end{aligned} \quad (51)$$

required in the Wick expansion of the interaction Hamiltonian. The property

$$a_p^s a_q^{+r} |0\rangle = \{a_p^s, a_q^{+r}\} |0\rangle = (2\pi)^3 \delta^3(\mathbf{p} - \mathbf{q}), \quad (52)$$

yields

$$\psi^+(x)|e^-, \mathbf{q}, r\rangle = \left(\frac{u_r(\mathbf{q})}{\sqrt{N_1}} \right) e^{-i\omega_1(\mathbf{q})x_0} e^{i\mathbf{q} \cdot \mathbf{x}}. \quad (53)$$

The resulting exponential contributes to the total energy-momentum conservation, with the physical energy $\omega_1(\mathbf{q})$ arising from the dispersion relations. A similar result can be obtained for the remaining spinors, thus validating the replacements shown in Eq. (50).

The amplitude \mathcal{M}_F for the graph shown in the left hand side of Fig. 2 is

$$\begin{aligned} i\mathcal{M}_F &= (-1) \frac{1}{N_1(\mathbf{p}_1)N_2(\mathbf{p}_2)} \bar{v}^r(p_2) (-ie\gamma^\mu) u^s(p_1) \\ &\quad D_{\mu\nu}(p) \int \frac{d^4q}{(2\pi)^4} \text{tr} \left[(-ie\gamma^\rho) S(q-p) (-ie\gamma^\nu) S(q) \right] \\ &\quad \times D_{\rho\sigma}(p) \bar{u}^s(p_1) (-ie\gamma^\sigma) v^r(p_2), \end{aligned} \quad (54)$$

where the minus sign comes from the fermion loop, and $p^\mu = p_1^\mu + p_2^\mu$. Let us define the currents

$$\begin{aligned} J_1^\mu(p_1, p_2) &= \frac{1}{\sqrt{N_1(\mathbf{p}_1)N_2(\mathbf{p}_2)}} \bar{v}^r(p_2) \gamma^\mu u^s(p_1), \\ J_2^\mu(p_1, p_2) &= \frac{1}{\sqrt{N_1(\mathbf{p}_1)N_2(\mathbf{p}_2)}} \bar{u}^s(p_1) \gamma^\mu v^r(p_2) \\ &= [J_1^\mu(p_1, p_2)]^*. \end{aligned} \quad (55)$$

Due to current conservation at the ingoing and outgoing vertices, $p_\mu J_1^\mu = 0$ and $p_\mu J_2^\mu = 0$, the only contribution from the photon propagator to the amplitude \mathcal{M}_F arises from the term containing $\eta_{\mu\nu}$.

In the center of mass frame ($\mathbf{p} = 0$) and using Eq. (48) we can write

$$\mathcal{M}_F = -\frac{e^4}{p^4} J_1^\nu J_2^\mu \int \frac{d^3q}{(2\pi)^4} \int_{C_{LW}} dq_0 (-i) \text{Tr} \left[\gamma_\mu \frac{(\gamma^0 f(q_0 - p_0) + \gamma^i q_i + m)}{D_{q-p}} \gamma_\nu \frac{(\gamma^0 f(q_0) + \gamma^i q_i + m)}{D_q} \right], \quad (56)$$

where

$$\begin{aligned} D_q &= g^2(q_0 - \omega_1)(q_0 - \omega_2)(q_0 - W_1) \\ &\quad \times (q_0 - W_2), \end{aligned} \quad (57)$$

$$\begin{aligned} D_{q-p} &= g^2(q_0 - \tilde{\omega}_1)(q_0 - \tilde{\omega}_2)(q_0 - \tilde{W}_1) \\ &\quad \times (q_0 - \tilde{W}_2), \end{aligned} \quad (58)$$

with

$$\begin{aligned} \tilde{\omega}_1 &= p_0 + \omega_1, \\ \tilde{\omega}_2 &= p_0 + \omega_2, \\ \tilde{W}_1 &= p_0 + W_1, \\ \tilde{W}_2 &= p_0 + W_2. \end{aligned} \quad (59)$$

It is convenient to define

$$\begin{aligned} T_{\mu\nu}(p_0, q_0, \mathbf{q}) &\equiv \text{Tr} \left[\gamma_\mu (\gamma^0 f(q_0 - p_0) + \gamma^i q_i + m) \gamma_\nu \right. \\ &\quad \left. \times (\gamma^0 f(q_0) + \gamma^i q_i + m) \right], \end{aligned} \quad (60)$$

and

$$I(p_0, q_0, \mathbf{q}) \equiv \frac{-i}{D_q D_{q-p}}, \quad (61)$$

together with

$$J_{\mu\nu}(p_0, \mathbf{q}) = \int_{C_{LW}} dq_0 T_{\mu\nu}(p_0, q_0, \mathbf{q}) I(p_0, q_0, \mathbf{q}). \quad (62)$$

We recall that the poles ω_1 and W_1 and W_2 are in lower complex p_0 -plane, while ω_2 is in the upper complex p_0 -plane.

We define the corresponding Lee-Wick contour C_{LW} , shown in Fig. 3., such that the poles ω_1 , W_1 , W_2 , $\tilde{\omega}_1$, \tilde{W}_1 and \tilde{W}_2 are in the lower sector, while the poles ω_2 and $\tilde{\omega}_2$ are in the upper sector. Then we have two ways of calculating the integral $J_{\mu\nu}(p_0, \mathbf{q})$ by closing the Lee-Wick contour in the upper or the lower complex p_0 -plane.

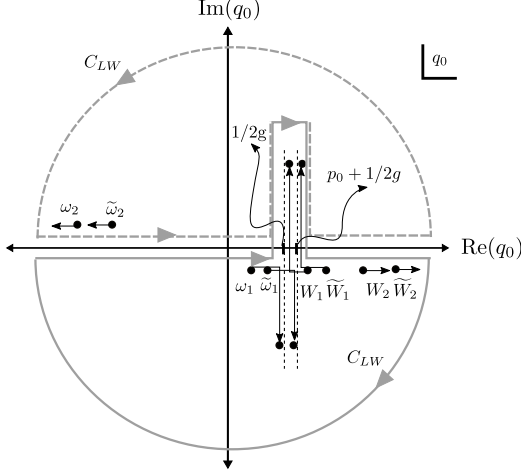


FIG. 3: The Lee-Wick contour C_{LW} used to compute the imaginary part of the forward scattering process \mathcal{M}_F . Also shown is the trajectory of each pole as the energy increases.

First, closing the contour downward yields

$$\begin{aligned}
 J_{\mu\nu}(p_0, \mathbf{q}) &= (-2\pi i) \sum_z T_{\mu\nu}(q_0, p_0, \mathbf{q})|_{q_0=z} \\
 &\times [\text{Res } I(q_0, p_0, \mathbf{q})]_{q_0=z} \equiv (-2\pi i) \\
 &\times \sum_z [T_{\mu\nu}(q_0, p_0, \mathbf{q})]_{q_0=z} I_z, \quad (63)
 \end{aligned}$$

where z runs over the poles $\omega_1, W_1, W_2, \tilde{\omega}_1, \tilde{W}_1$ and \tilde{W}_2 and $[\text{Res } I(q_0, \dots)]_{q_0=z}$ denotes the residue of $I(q_0, \dots)$ at the pole z . Since the integral $J_{\mu\nu}(p_0, \mathbf{q})$ in a full circle at infinite is zero because the integrand behaves as q_0^{-4} in that limit, closing the Lee-Wick contour upward including the remaining poles $\omega_2, \tilde{\omega}_2$, should yield the same result as Eq. (63). In other words, we expect

$$\begin{aligned}
 J_{\mu\nu}(p_0, \mathbf{q}) &= (2\pi i) \sum_{\bar{z}} T_{\mu\nu}(q_0, p_0, \mathbf{q})|_{q_0=\bar{z}} \\
 &\times [\text{Res } I(q_0, p_0, \mathbf{q})]_{q_0=\bar{z}} \equiv (2\pi i) \\
 &\times \sum_{\bar{z}} T_{\mu\nu}(q_0, p_0, \mathbf{q})|_{q_0=\bar{z}} I_{\bar{z}}, \quad (64)
 \end{aligned}$$

where \bar{z} runs over the poles $\omega_2, \tilde{\omega}_2$. Both $I_z, I_{\bar{z}}$ are functions of p_0 and \mathbf{q} , which we do not write in the following to keep the notation simple.

Calculating I_z and $I_{\bar{z}}$ according to the definitions in

Eqs. (63) and (64), respectively, we obtain

$$\begin{aligned}
 I_{\omega_1} &= \frac{\pi}{g^2 E(\mathbf{q}) N_1 p_0 (\omega_1 - \tilde{\omega}_2) (\omega_1 - \tilde{W}_1) (\omega_1 - \tilde{W}_2)}, \\
 I_{\tilde{\omega}_1} &= \frac{-\pi}{g^2 E(\mathbf{q}) N_1 p_0 (\tilde{\omega}_1 - \omega_2) (\tilde{\omega}_1 - W_1) (\tilde{\omega}_1 - W_2)}, \\
 I_{W_1} &= \frac{-\pi}{g^2 E(\mathbf{q}) N_1 p_0 (W_1 - \tilde{\omega}_1) (\tilde{W}_1 - \omega_2) (\tilde{W}_1 - W_2)}, \\
 I_{\tilde{W}_1} &= \frac{\pi}{g^2 E(\mathbf{q}) N_1 p_0 (\tilde{W}_1 - \omega_1) (\tilde{W}_1 - \omega_2) (\tilde{W}_1 - W_2)}, \\
 I_{W_2} &= \frac{\pi}{g^2 E(\mathbf{q}) N_2 p_0 (W_2 - \tilde{\omega}_1) (W_2 - \tilde{\omega}_2) (W_2 - \tilde{W}_1)}, \\
 I_{\tilde{W}_2} &= \frac{-\pi}{g^2 E(\mathbf{q}) N_2 p_0 (\tilde{W}_2 - \omega_1) (\tilde{W}_2 - \omega_2) (\tilde{W}_2 - W_1)}, \\
 I_{\omega_2} &= \frac{\pi}{g^2 E(\mathbf{q}) N_2 p_0 (\omega_2 - \tilde{\omega}_1) (\omega_2 - \tilde{W}_1) (\omega_2 - \tilde{W}_2)}, \\
 I_{\tilde{\omega}_2} &= \frac{-\pi}{g^2 E(\mathbf{q}) N_2 p_0 (\tilde{\omega}_2 - \omega_1) (\tilde{\omega}_2 - W_1) (\tilde{\omega}_2 - W_2)}, \quad (65)
 \end{aligned}$$

where we have repeatedly used Eqs. (38) and (42). In the equations above, the eigenvalues ω_1, ω_2, W_1 and W_2 are all functions of $E(\mathbf{q})$, according to the definitions (5) and (6).

In order to compute the imaginary part of \mathcal{M}_F we recall that

$$\mathcal{M}_F = -\frac{e^4}{p^4} J_1^\nu J_2^\mu \int \frac{d^3 q}{(2\pi)^4} \sum_z T_{\mu\nu}(q_0, p_0, \mathbf{q})|_{q_0=z} I_z, \quad (66)$$

and we use the relation

$$\begin{aligned}
 \text{Im} \mathcal{M}_F(p_0 + i\epsilon) &= \frac{1}{2i} \text{Disc}(\mathcal{M}_F) \\
 &\equiv \frac{1}{2i} (\mathcal{M}_F(p_0 + i\epsilon) - \mathcal{M}_F(p_0 - i\epsilon)). \quad (67)
 \end{aligned}$$

The discontinuity in Eq. (66) arises only from each contribution $I_z, I_{\bar{z}}$. Assuming that one of them occurs at $p_0 = \alpha$, we focus on the relevant part of I_z which we write as

$$I_z = U(p_0, \mathbf{q}, \dots) \frac{1}{(p_0 - \alpha)}. \quad (68)$$

The contribution to the discontinuity will be

$$\begin{aligned}
 \text{Disc}(I_z) &= U(p_0 = \alpha, \mathbf{q}) \frac{1}{2i} \\
 &\times \left(\frac{1}{p_0 - \alpha + i\epsilon} - \frac{1}{p_0 - \alpha - i\epsilon} \right) \\
 &= \pi U(p_0 = \alpha, \mathbf{q}) \delta(p_0 - \alpha), \quad (69)
 \end{aligned}$$

according to the identity

$$\frac{\epsilon}{x^2 + \epsilon^2} = \pi \delta(x). \quad (70)$$

Next we list the contributions to the discontinuity arising from each I_z and leave to the Appendix A the detailed derivation of the results

$$\begin{aligned}
Disc(I_{\omega_1}) &= \frac{i\pi^2}{E^2(\mathbf{q})N_1N_2} \delta(p_0 - (\omega_1 - \omega_2)), \\
Disc(I_{\tilde{\omega}_1}) &= \frac{-i\pi^2}{E^2(\mathbf{q})(N_1)^2} \delta(p_0 - (W_1 - \omega_1)), \\
Disc(I_{W_1}) &= \frac{i\pi^2}{E^2(\mathbf{q})(N_1)^2} \delta(p_0 - (W_1 - \omega_1)), \\
Disc(I_{\tilde{W}_1}) &= \frac{-i\pi^2}{E^2(\mathbf{q})N_1N_2} \delta(p_0 - (\omega_1 - \omega_2)), \\
Disc(I_{W_2}) &= \frac{i\pi^2}{E^2(\mathbf{q})N_1N_2} \delta(p_0 - (\omega_1 - \omega_2)), \\
Disc(I_{\tilde{W}_2}) &= 0, \\
Disc(I_{\omega_2}) &= 0, \\
Disc(I_{\tilde{\omega}_2}) &= \frac{i\pi^2}{E^2(\mathbf{q})N_1N_2} \delta(p_0 - (\omega_1 - \omega_2)). \quad (71)
\end{aligned}$$

The next step is to calculate the discontinuities in \mathcal{M} , from the general expression

$$\begin{aligned}
Disc(\mathcal{M}_F) &= -\frac{e^4}{p^4} J_1^\mu(p_1) J_2^\nu(p_2) \int \frac{d^3q}{(2\pi)^4} \\
&\times \sum_z T_{\mu\nu}(q_0, p_0, \mathbf{q}) \Big|_{q_0=z} Disc(I_z). \quad (72)
\end{aligned}$$

Next we concentrate on the basic ingredient

$$U_{\mu\nu}(\mathbf{q}) \Big|_z \equiv T_{\mu\nu}(q_0, p_0, \mathbf{q}) \Big|_{q_0=z} \times Disc(I_z). \quad (73)$$

where we introduce the further notation

$$\begin{aligned}
T_{\mu\nu}(q_0, p_0, \mathbf{q}) &= Tr(\gamma_\mu(\gamma^0 f(q_0 - p_0) + \gamma^i q_i + m) \\
&\gamma_\nu(\gamma^0 f(q_0) + \gamma^i q_i + m)) \equiv F_{\mu\nu}(f(q_0 - p_0), f(q_0)),
\end{aligned} \quad (74)$$

in order to emphasize the relevant variables at this stage.

According to Eq. (72) together with the definitions (73) and (74), in order to calculate $Disc(\mathcal{M}_F)$ we need the combinations $F_{\mu\nu}(f(q_0 - p_0), f(q_0)) \times Disc(I_z)$. The relevant terms here are the products of $F_{\mu\nu}(f(q_0 - p_0), f(q_0))$ times the delta functions appearing in each $Disc(I_z)$, arising from Eqs. (71). We provide a list of them in the following equations

$$\begin{aligned}
F_{\mu\nu}(f(q_0 - p_0), f(q_0)) \Big|_{q_0=\omega_1} \delta(p_0 - (\omega_1 - \omega_2)) &= F_{\mu\nu}(-E(\mathbf{q}), E(\mathbf{q})) \delta(p_0 - (\omega_1 - \omega_2)), \\
F_{\mu\nu}(f(q_0 - p_0), f(q_0)) \Big|_{q_0=\tilde{\omega}_1} \delta(p_0 - (W_1 - \omega_1)) &= F_{\mu\nu}(E(\mathbf{q}), E(\mathbf{q})) \delta(p_0 - (W_1 - \omega_1)), \\
F_{\mu\nu}(f(q_0 - p_0), f(q_0)) \Big|_{q_0=W_1} \delta(p_0 - (W_1 - \omega_1)) &= F_{\mu\nu}(E(\mathbf{q}), E(\mathbf{q})) \delta(p_0 - (W_1 - \omega_1)), \\
F_{\mu\nu}(f(q_0 - p_0), f(q_0)) \Big|_{q_0=\tilde{W}_1} \delta(p_0 - (\omega_1 - \omega_2)) &= F_{\mu\nu}(E(\mathbf{q}), -E(\mathbf{q})) \delta(p_0 - (\omega_1 - \omega_2)), \\
F_{\mu\nu}(f(q_0 - p_0), f(q_0)) \Big|_{q_0=W_2} \delta(p_0 - (\omega_1 - \omega_2)) &= F_{\mu\nu}(E(\mathbf{q}), -E(\mathbf{q})) \delta(p_0 - (\omega_1 - \omega_2)), \\
F_{\mu\nu}(f(q_0 - p_0), f(q_0)) \Big|_{q_0=\tilde{\omega}_2} \delta(p_0 - (\omega_1 - \omega_2)) &= F_{\mu\nu}(-E(\mathbf{q}), E(\mathbf{q})) \delta(p_0 - (\omega_1 - \omega_2)). \quad (75)
\end{aligned}$$

In proving the above results we have extensively used the relations (8). In this way, the contributions to the discontinuity of \mathcal{M} are

$$\begin{aligned}
U_{\mu\nu}(\mathbf{q}) \Big|_{\omega_1} &= \frac{i\pi^2}{E^2(\mathbf{q})N_1N_2} F_{\mu\nu}(-E(\mathbf{q}), E(\mathbf{q})) \\
&\times \delta(p_0 - (\omega_1 - \omega_2)), \quad (76)
\end{aligned}$$

$$\begin{aligned}
U_{\mu\nu}(\mathbf{q}) \Big|_{\tilde{\omega}_1} &= -\frac{i\pi^2}{E^2(\mathbf{q})(N_1)^2} F_{\mu\nu}(E(\mathbf{q}), E(\mathbf{q})) \\
&\times \delta(p_0 - (W_1 - \omega_1)), \quad (77)
\end{aligned}$$

$$\begin{aligned}
U_{\mu\nu}(\mathbf{q}) \Big|_{W_1} &= \frac{i\pi^2}{E^2(\mathbf{q})(N_1)^2} F_{\mu\nu}(E(\mathbf{q}), E(\mathbf{q})) \\
&\times \delta(p_0 - (W_1 - \omega_1)), \quad (78)
\end{aligned}$$

$$\begin{aligned}
U_{\mu\nu}(\mathbf{q}) \Big|_{\tilde{W}_1} &= -\frac{i\pi^2}{E^2(\mathbf{q})N_1N_2} F_{\mu\nu}(E(\mathbf{q}), -E(\mathbf{q})) \\
&\times \delta(p_0 - (\omega_1 - \omega_2)), \quad (79)
\end{aligned}$$

$$U_{\mu\nu}(\mathbf{q})\Big|_{W_2} = \frac{i\pi^2}{E^2(\mathbf{q})N_1N_2}F_{\mu\nu}(E(\mathbf{q}), -E(\mathbf{q})) \times \delta(p_0 - (\omega_1 - \omega_2)), \quad (80)$$

$$U_{\mu\nu}(\mathbf{q})\Big|_{\tilde{W}_2} = 0, \quad (81)$$

$$U_{\mu\nu}(\mathbf{q})\Big|_{\omega_2} = 0, \quad (82)$$

$$U_{\mu\nu}(\mathbf{q})\Big|_{\tilde{\omega}_2} = \frac{i\pi^2}{E^2(\mathbf{q})N_1N_2}F_{\mu\nu}(-E(\mathbf{q}), E(\mathbf{q})) \times \delta(p_0 - (\omega_1 - \omega_2)). \quad (83)$$

Let us observe that unexpected cancellations occur

$$\begin{aligned} U_{\mu\nu}(\mathbf{q})\Big|_{\tilde{\omega}_1} + U_{\mu\nu}(\mathbf{q})\Big|_{W_1} &= 0, \\ U_{\mu\nu}(\mathbf{q})\Big|_{\tilde{W}_1} + U_{\mu\nu}(\mathbf{q})\Big|_{W_2} &= 0. \end{aligned} \quad (84)$$

Now we are in position to calculate the final result for $Disc(\mathcal{M}_F)$, in agreement with the Lee-Wick contour C_{LW} previously chosen. When we evaluate the integral over q_0 in $J_{\mu\nu}(p_0, \mathbf{q})$, Eq. (62), closing C_{LW} from below, we get the contributions from the poles $z : \omega_1, W_1$ and W_2 plus those obtained by the displacement in p_0 , according to Eq. (63). This means that only the corresponding contributions from $U_{\mu\nu}(\mathbf{q})\Big|_z$ add up in Eq. (72). Considering further that $U_{\mu\nu}(\mathbf{q})\Big|_{\tilde{W}_2} = 0$ together with the cancellations in Eq. (84), the final contribution to $Disc(\mathcal{M}_F)$ comes only from $U_{\mu\nu}(\mathbf{q})\Big|_{\omega_1}$, with the result

$$\begin{aligned} \frac{1}{i}Disc(\mathcal{M}_F) &= \frac{e^4}{p^4} J_1^\nu J_2^\mu \int \frac{d^3q}{(2\pi)^2} \frac{1}{2E(\mathbf{q})N_1 2E(\mathbf{q})N_2} \\ &\quad Tr(\gamma_\mu(\gamma^0 E(\mathbf{q}) - \gamma^i q_i - m) \\ &\quad \gamma_\nu(\gamma^0 E(\mathbf{q}) + \gamma^i q_i + m)) \\ &\quad \times \delta(p_0 - (\omega_1 - \omega_2)), \end{aligned} \quad (85)$$

in the center of mass frame. If we were to close the Lee-Wick contour from above in Eq. (62), the relation (64) tells us that nothing but the poles $\tilde{z} : \omega_2, \tilde{\omega}_2$ need to be included. Then, the corresponding contributions to $U_{\mu\nu}(\mathbf{q})\Big|_{\tilde{z}}$ arise only from $U_{\mu\nu}(\mathbf{q})\Big|_{\tilde{\omega}_2}$ in Eq. (83), which is equal to $U_{\mu\nu}(\mathbf{q})\Big|_{\omega_1}$. In this way, we have explicitly

shown that the result in Eq. (85) is independent of the way in which we calculate the q_0 integral in Eq. (62). As mentioned previously, this is a consequence of the fact that such integral is zero in a circle at infinity.

IV. VERIFICATION OF THE OPTICAL THEOREM

We have already calculated the left hand side of Eq. (44) in the evaluation of the optical theorem. Now we deal with the contribution of the final states required in the right hand side of this equation. To the order considered we have only two-particle final states. In this way we start by calculating the amplitude \mathcal{M}_I for the process $e^-(p_1, s) + e^+(p_2, r) \rightarrow e^-(k_1, \bar{s}) + e^+(k_2, \bar{r})$. As already stated, we apply the Lee-Wick prescription in such a way that we only consider the asymptotic states corresponding to those with positive-metric, corresponding to the frequencies ω_1 and ω_2 . We obtain

$$\mathcal{M}_I = -\frac{ie}{p^2} J_1^\mu [\bar{v}^{\bar{r}}(k_2)(-ie\gamma_\mu)u^{\bar{s}}(k_1)] \frac{1}{\sqrt{N_1(\mathbf{k}_1)N_2(\mathbf{k}_2)}}, \quad (86)$$

where we have introduced the current defined in Eq. (55). This yields

$$|\mathcal{M}_I|^2 = \frac{e^2}{p^4} \frac{1}{N_1(\mathbf{k}_1)N_2(\mathbf{k}_2)} J_1^\mu(p_1, p_2) [J_1^\alpha(p_1, p_2)]^* [\bar{v}^{\bar{r}}(k_2)(e\gamma_\mu)u^{\bar{s}}(k_1)] \bar{u}^{\bar{s}}(k_1)(e\gamma_\alpha)v^{\bar{r}}(k_2). \quad (87)$$

Recalling the right hand side of Eq. (44), which we denote by W ,

$$\begin{aligned} W &= \sum_{\bar{r}, \bar{s}} \int \frac{d^3k_1}{(2\pi)^3} \frac{1}{2E(\mathbf{k}_1)} \frac{d^3k_2}{(2\pi)^3} \frac{1}{2E(\mathbf{k}_2)} (2\pi)^4 \\ &\quad \times \delta^4(p = k_1 + k_2) |\mathcal{M}_I|^2, \end{aligned} \quad (88)$$

we perform the sum over the spin components \bar{s}, \bar{r} with the result

$$\begin{aligned} \sum_{\bar{r}, \bar{s}} |\mathcal{M}_I|^2 &= \frac{1}{p^4} \frac{1}{N_1(\mathbf{k}_1)N_2(\mathbf{k}_2)} J_1^\mu(p_1, p_2) [J_1^\alpha(p_1, p_2)]^* \\ &\quad Tr[\gamma_\mu(\gamma k_1 + m)\gamma_\alpha(\gamma k_2 - m)]. \end{aligned} \quad (89)$$

In the center of mass frame we have

$$\begin{aligned} W &= \frac{e^4}{p^4} J_1^\mu(p_1, p_2) [J_1^\alpha(p_1, p_2)]^* \int \frac{d^3k_1 d^3k_2}{(2\pi)^2 2E(\mathbf{k}_1)N_1(\mathbf{k}_1) 2E(\mathbf{k}_2)N_2(\mathbf{k}_2)} \delta^3(\mathbf{k}_1 + \mathbf{k}_2) \delta(p_0 - (\omega_1 - \omega_2)) \\ &\quad \times Tr[\gamma_\mu(\gamma^0 E(\mathbf{k}_1) + \gamma^i k_{1i} + m)\gamma_\alpha(\gamma^0 E(\mathbf{k}_2) + \gamma^i k_{2i} - m)]. \end{aligned} \quad (90)$$

In the last step we relabel $\mathbf{k}_1 \rightarrow \mathbf{q}$ and we integrate over d^3k_2 . In this way

$$W = \frac{e^4}{p^4} J_1^\mu(p_1, p_2) [J_1^\alpha(p_1, p_2)]^* \int \frac{d^3q}{(2\pi)^2 2E(\mathbf{q}) N_1(\mathbf{q}) 2E(\mathbf{q}) N_2(\mathbf{q})} \delta(p_0 - (\omega_1(\mathbf{q}) - \omega_2(\mathbf{q}))) \\ \times \text{Tr} [\gamma_\mu (\gamma^0 E(\mathbf{q}) + \gamma^i q_i + m) \gamma_\alpha (\gamma^0 E(\mathbf{q}) - \gamma^i q_i - m)]. \quad (91)$$

The cyclic property of the trace together with the relation $[J_1^\alpha]^* = J_2^\alpha$ from Eq. (55) show that $W = 2\text{Im}[\mathcal{M}_F] = -i\text{Disc}(\mathcal{M}_F)$, where the last expression is given in Eq. (85), thus verifying the optical theorem.

The effective nature of our model restrict us to consider external physical states with real energy p_0 , meaning to impose the limit $E(p) < 1/4g$ in the center of mass frame. However, for intermediate states this no longer applies allowing the momentum integration to go beyond this critical energy which introduces complex poles as shown in Fig.(3). Nevertheless, the optical theorem still holds in this case since $\delta(p_0 - (\omega_1 - \omega_2))$, which dominates both sides in Fig. 2, picks up an imaginary part from ω_1 , thus being zero on both sides.

V. CONCLUSIONS AND OUTLOOK

The effective approach to quantum field theory provides a powerful tool to search for new physics beyond the standard model. In particular, the search for quantum gravity effects at low energies in the form of Lorentz symmetry violations has become an active research area from both the phenomenological and experimental points of view. The majority of these searches have been realized by coupling constant tensors, yielding Lorentz invariance violation, with derivative operators of renormalizable mass dimension. In this way one guarantees from the beginning some crucial requirements about stability and unitarity in the effective quantum field theory.

However, the study of Lorentz symmetry violation incorporating higher-order derivative operators has attracted interest in the last years. There are good reasons for this: (i) bounds arising from higher-order derivative operators have been less explored experimentally as compared to those arising from renormalizable models and (ii) due to the increase of the number of degrees of freedom in models with higher-order derivative operators, they have the potentiality to capture higher-energy degrees of freedom associated to new physics. Also, it is well known that the introduction of higher-order derivative operators has the advantage to smooth ultraviolet divergencies. The nonminimal SME and the Myers-Pospelov model provide the framework to detect possible effects of higher-order Lorentz-invariance violation. One of the main goals of this work is to emphasize that when dealing with models described by operators with mass dimension greater than four, it is also necessary to consider the consequences upon unitarity, as shown in the past works of Lee and Wick.

The Lee-Wick extension of quantum electrodynamics is a modified quantum field theory with an indefinite metric. One of the goals in this construction has been to prove the preservation of unitarity by applying what is now called the Lee-Wick prescription [18]. This prescription consists in removing the negative-metric states from the asymptotic Hilbert space and in redefining the contour integration in Feynman diagrams to preserve the perturbative unitarity of the S matrix. The prescription has to be defined order by order in the perturbative series expansion.

In this work we have followed the Lee-Wick prescription in a model where Lorentz violation is explicitly broken with a preferred four vector coupled to a higher dimension derivative operator. In particular we have focussed in the dimension-five operators of the Myers-Pospelov model. Our goal has been to test the Lee-Wick prescription in order to verify perturbative unitarity in the form of the optical theorem. We have found that unitarity is preserved at the one loop level in the annihilation channel of the scattering process $e^+(p_2, r) + e^-(p_1, s)$. The computation has taken into consideration the discontinuities of the Feynman diagram for energies below and above the critical energy $E = 1/4g$.

A further step in generalizing this result will be to probe the optical theorem in each of the remaining one-loop Feynman diagram that appear in the model, for the same scattering processes.

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Appendix A: THE CONTRIBUTIONS TO THE DISCONTINUITY OF $I(p_0, \mathbf{q})$

For each term I_z or $I_{\bar{z}}$ we indicate the possible contributions to the discontinuity (the choices of $p_0 = Y(|\mathbf{q}|)$ which make zero each denominators) and analyze which

of such conditions can be fulfilled. On the one hand we have

$$\begin{aligned} p_0(|\mathbf{p}|) &= \omega_1(|\mathbf{p}|) + |\omega_2(|\mathbf{p}|)|, \\ &= \frac{\sqrt{1+4gE(\mathbf{p})} - \sqrt{1-4gE(\mathbf{p})}}{2g}, \\ &= \frac{N_2(|\mathbf{p}|) - N_1(|\mathbf{p}|)}{2g}, \end{aligned} \quad (\text{A1})$$

in the center of mass frame where $\mathbf{p}_{e+} = -\mathbf{p}_{e-} = \mathbf{p}$. On the other hand, the condition for a discontinuity to occur is that the equation $p_0(|\mathbf{p}|) = Y(|\mathbf{q}|)$ has a solution for $|\mathbf{q}|$. The function $Y(|\mathbf{q}|)$ will depend on the various combinations of the eigenvalues ω_1, ω_2, W_1 and W_2 defined in Eqs. (5) and (6). We have to consider only the positive contributions to p_0 , which we discuss in the following, in order to further evaluate the discontinuities arising from Eqs. (65).

1. Identification of the contributions to the discontinuity

The following cases arise: (1)

$$p_0 = \frac{N_2(\mathbf{q}) - N_1(\mathbf{q})}{2g}, \quad (\text{A2})$$

which is directly solved by choosing $\mathbf{q} = \mathbf{p}$ according to Eq. (A1).

(2)

$$p_0 = \frac{N_2(\mathbf{q}) + N_1(\mathbf{q})}{2g} = \frac{\sqrt{1+4gE(\mathbf{q})} + \sqrt{1-4gE(\mathbf{q})}}{2g}. \quad (\text{A3})$$

Substituting Eq. (A1) and taking the square of the resulting equation we obtain

$$\sqrt{1-16g^2E^2(\mathbf{p})} = -\sqrt{1-16g^2E^2(\mathbf{q})}, \quad (\text{A4})$$

which produces a sign inconsistency, leading to no solution in this case.

(3)

$$p_0 = \frac{N_2(\mathbf{q})}{g} = \frac{\sqrt{1+4gE(\mathbf{q})}}{g}. \quad (\text{A5})$$

Replacing p_0 as before and taking the square of the resulting equation yields

$$-\sqrt{1-16g^2E^2(\mathbf{p})} = 1 + 8gE^2(\mathbf{q}). \quad (\text{A6})$$

The left hand side of the above equation is negative, while the right hand side is positive, leading again to no solution for $|\mathbf{q}|$.

(4)

$$p_0 = \frac{N_1(\mathbf{q})}{g} = \frac{\sqrt{1-4gE(\mathbf{q})}}{g}. \quad (\text{A7})$$

Replacing p_0 as before and taking the square of the resulting equation yields

$$-\sqrt{1-16g^2E^2(\mathbf{p})} = 1 - 8gE^2(\mathbf{q}). \quad (\text{A8})$$

Since $4gE^2(\mathbf{q}) < 1$ we still can have a solution in the region

$$1 < 8gE^2(\mathbf{q}) < 2. \quad (\text{A9})$$

In this case the right hand side of Eq. (A8) is negative. Solving for the resulting equation we get

$$E^2(\mathbf{q}) = \frac{1 + \sqrt{1-16g^2E^2(\mathbf{p})}}{8g}. \quad (\text{A10})$$

In fact, Eq. (A9) is satisfied for the whole range of values of $E(\mathbf{p})$. For $E(\mathbf{p}) = 0$, we have $8gE^2(\mathbf{q}) = 1 + \sqrt{1-16g^2m^2} < 2$, while for $E(\mathbf{p}) = E_{\max} = 1/4g$ we obtain $8gE^2(\mathbf{q}) = 1$. Thus this case will contribute to the discontinuity.

2. The particular cases

To compute $Disc(I_{\omega_1})$ from the first Eq. (65) we have the possible choices for p_0

$$\begin{aligned} p_0 &= \omega_1 - \omega_2 = \frac{N_2 - N_1}{2g}, \\ p_0 &= \omega_1 - W_1 = -\frac{N_1}{g} < 0, \\ p_0 &= \omega_1 - W_2 = -\frac{N_1 + N_2}{2g} < 0. \end{aligned} \quad (\text{A11})$$

where we have used Eqs. (5) and (6). Since $N_2 > N_1 > 0$ the only contribution arises only from the first case in Eqs. (A11), which yields

$$\begin{aligned} Disc(I_{\omega_1}) &= i(2\pi)^2 \frac{\delta(-\omega_1 + \omega_2 + p_0)}{g^2 2E(\mathbf{q})N_1} \\ &\times \frac{1}{p_0(\omega_1 - W_1 - p_0)(\omega_1 - W_2 - p_0)}. \end{aligned} \quad (\text{A12})$$

The delta function allows us to rewrite the denominator as

$$\frac{-1}{g^2(\omega_2 - \omega_1)(\omega_2 - W_1)(\omega_2 - W_2)}. \quad (\text{A13})$$

Using the relation (42) we finally arrive to

$$Disc(I_{\omega_1}) = \frac{(i\pi^2)}{E^2(\mathbf{q})N_1N_2} \delta(p_0 - (\omega_1 - \omega_2)). \quad (\text{A14})$$

The next calculation for $Disc(I_{\omega_1+p_0})$ follows closely the previous case, so that we mention only the relevant

points. From Eq. (65) we read the following possible values for p_0

$$\begin{aligned} p_0 &= -(\omega_1 - \omega_2) = -\frac{N_2 - N_1}{g} < 0, \\ p_0 &= -(\omega_1 - W_1) = \frac{N_1}{g}, \\ p_0 &= -(\omega_1 - W_2) = \frac{N_1 + N_2}{2g}, \end{aligned} \quad (\text{A15})$$

According to Section (A1) only the second case in Eqs. (A15) survives, yielding

$$Disc(I_{\omega_1+p_0}) = -\frac{i\pi^2}{E^2(\mathbf{q})(N_1)^2} \delta(p_0 - (W_1 - \omega_1)). \quad (\text{A16})$$

For $Disc(I_{W_1})$ the possibilities for p_0 are

$$\begin{aligned} p_0 &= W_1 - \omega_1 = \frac{N_1}{g}, \\ p_0 &= W_1 - \omega_2 = \frac{N_1 + N_2}{2g}, \\ p_0 &= W_1 - W_2 = \frac{N_1 - N_2}{2g} < 0. \end{aligned} \quad (\text{A17})$$

From Section (A1) we conclude that the only contribution arises from the first case in Eqs. (A17), which produces

$$Disc(I_{W_1}) = \frac{i\pi^2}{E^2(\mathbf{q})(N_1)^2} \delta(p_0 - (W_1 - \omega_1)). \quad (\text{A18})$$

Now we look at $Disc(I_{\tilde{W}_1})$. From Eq. (65) we have the following possible values for p_0

$$\begin{aligned} p_0 &= -(W_1 - \omega_1) = -\frac{N_1}{g} < 0, \\ p_0 &= -(W_1 - \omega_2) = -\frac{N_1 + N_2}{2g} < 0, \\ p_0 &= -(W_1 - W_2) = \frac{N_2 - N_1}{2g}. \end{aligned} \quad (\text{A19})$$

From Section (A1), we conclude that the only contribution arises from the third term in Eqs. (A19). We are left with

$$Disc(I_{\tilde{W}_1}) = -\frac{i\pi^2}{E^2(\mathbf{q})N_1N_2} \delta(p_0 - (W_2 - W_1)). \quad (\text{A20})$$

From Eq. (7) we get $W_2 - W_1 = \omega_1 - \omega_2$ so that we can write

$$Disc(I_{\tilde{W}_1}) = -\frac{i\pi^2}{E^2(\mathbf{q})N_1N_2} \delta(p_0 - (\omega_1 - \omega_2)). \quad (\text{A21})$$

For $Disc(I_{W_2})$ the choices for p_0 are

$$\begin{aligned} p_0 &= W_2 - \omega_1 = \frac{N_2 + N_1}{2g}, \\ p_0 &= W_2 - \omega_2 = \frac{N_2}{g}, \\ p_0 &= W_2 - W_1 = \frac{N_2 - N_1}{2g}. \end{aligned} \quad (\text{A22})$$

The discontinuity arises only from the third contribution of the above equations yielding

$$Disc(I_{W_2}) = \frac{i\pi^2}{E^2(\mathbf{q})N_2N_1} \delta(p_0 - (\omega_1 - \omega_2)). \quad (\text{A23})$$

For $Disc(I_{\tilde{W}_2})$ we have

$$\begin{aligned} p_0 &= -(W_2 - \omega_1) = -\frac{N_2 + N_1}{2g} < 0, \\ p_0 &= -(W_2 - \omega_2) = -\frac{N_2}{g}, \\ p_0 &= -(W_2 - W_1) = -\frac{N_2 - N_1}{2g} < 0, \end{aligned} \quad (\text{A24})$$

in such a way that $Disc(I_{\tilde{W}_2})=0$.

For $Disc(I_{\omega_2})$ we have

$$\begin{aligned} p_0 &= \omega_2 - \omega_1 = \frac{(N_1 - N_2)}{2g} < 0, \\ p_0 &= \omega_2 - W_1 = -\frac{(N_1 + N_2)}{2g} < 0, \\ p_0 &= \omega_2 - W_2 = -\frac{N_2}{g} < 0. \end{aligned} \quad (\text{A25})$$

Since all the contributions are negative we conclude that $Disc(I_{\omega_2})=0$.

For $Disc(I_{\tilde{\omega}_2})$ we have

$$\begin{aligned} p_0 &= -(\omega_2 - \omega_1) = \frac{(N_2 - N_1)}{2g}, \\ p_0 &= -(\omega_2 - W_1) = \frac{(N_1 + N_2)}{2g}, \\ p_0 &= -(\omega_2 - W_2) = \frac{N_2}{g} < 0. \end{aligned} \quad (\text{A26})$$

Only the first term in the previous equations contribute, yielding

$$Disc(I_{\tilde{\omega}_2}) = \frac{i\pi^2}{E^2(\mathbf{q})N_1N_2} \delta(p_0 - (\omega_1 - \omega_2)). \quad (\text{A27})$$

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